# OpenMP Reduction Case Study: Trapezoid Integration Example

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## Find the area under the curve y = sin(x) for 0 ≤ x ≤ π using the Trapezoid Rule


Exact answer: $\int_0^{\pi} (\sin x) dx = -\cos x \,|_0^{\pi} = 2.0$



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## Don't do it this way !

```
const  double A  =  0.;
const  double B  =  M_PI;
double dx = ( B - A ) / (float) ( numSubdivisions – 1 );
double sum = (  Function( A ) + Function( B )  )  /  2.;

omp_set_num_threads(  numThreads );
#pragma  omp  parallel  for  default(none), shared(dx,sum)
for( int i = 1; i < numSubdivisions - 1; i++ )
{
        double x = A + dx * (float) i;
        double f = Function( x );
        sum  +=  f;
}
sum *= dx;
```

- There is no guarantee when each thread will execute this line
- There is not even a guarantee that each thread will finish this line before some other thread interrupts it.

| Assembly code: |
|---|
| Load sum<br>Add f<br>Store sum |

What if the scheduler decides to switch threads right here?



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## The answer should be 2.0 *exactly*, but in 30 trials, it's not even close. And, the answers aren't even consistent. How do we fix this?

| | |
|---|---|
| 0.469635 | 0.398893 |
| 0.517984 | 0.446419 |
| 0.438868 | 0.431204 |
| 0.437553 | 0.501783 |
| 0.398761 | 0.334996 |
| 0.506564 | 0.484124 |
| 0.489211 | 0.506362 |
| 0.584810 | 0.448226 |
| 0.476670 | 0.434737 |
| 0.530668 | 0.444919 |
| 0.500062 | 0.442432 |
| 0.672593 | 0.548837 |
| 0.411158 | 0.363092 |
| 0.408718 | 0.544778 |
| 0.523448 | 0.356299 |



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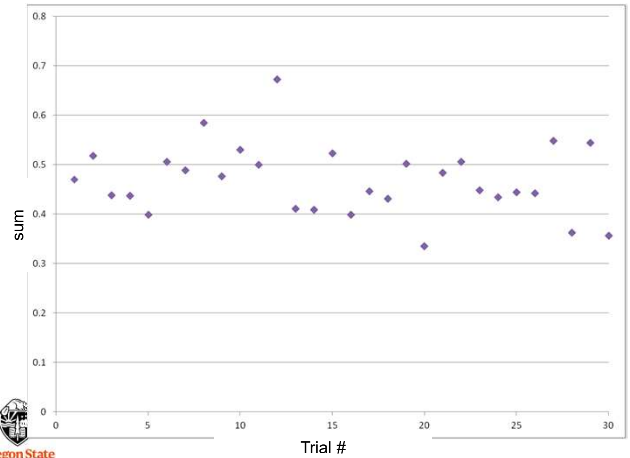






## There are Three Ways to Make the Summing Work Correctly: #1: Atomic



**1**

```
#pragma omp parallel for shared(dx)
for( int i = 0; i < numSubdivisions; i++ )
{
        double x = A + dx * (float) i;
        double f = Function( x );
        #pragma omp atomic
        sum += f;
}
```

- More lightweight than *critical* (#2)
- Uses a hardware instruction CMPXCHG (compare-and-exchange)
- Can only handle these operations:

  x++, ++x, x--, --x
  x op= expr , x = x op expr , x = expr op x
  where op is one of: +, -, *, /, &, |, ^, <<, >>





## There are Three Ways to Make the Summing Work Correctly: #2: Critical



**2**

```
#pragma omp parallel for shared(dx)
for( int i = 0; i < numSubdivisions; i++ )
{
        double x = A + dx * (float) i;
        double f = Function( x );
        #pragma omp critical
        sum += f;
}
```

- More heavyweight than *atomic* (#1)
- Allows only one thread at a time to enter this block of code (similar to a mutex)
- Can have any operations you want in this block of code





## There are Three Ways to Make the Summing Work Correctly: #3: Reduction



**3**

```
#pragma omp parallel for shared(dx),reduction(+:sum)
for( int i = 0; i < numSubdivisions; i++ )
{
        double x = A + dx * (float) i;
        double f = Function( x );
        sum += f;
}
```

- OpenMP creates code to make this as fast as possible
- Reduction operators can be: + , - , * , & , | , ^ , && , || , max , min

## Speed of Reduction vs. Atomic vs. Critical

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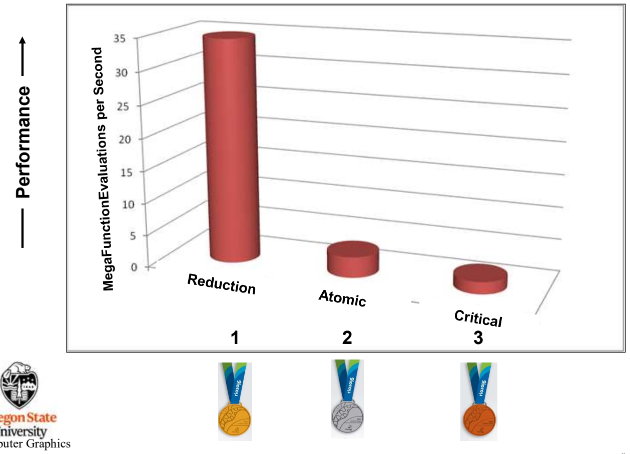
(*up* = faster)

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## So, do it this way !

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```
const double A = 0.;
const double B = M_PI;

double dx = ( B - A ) / (float) ( numSubdivisions – 1 );

omp_set_num_threads( numThreads );

double sum = ( Function( A ) + Function( B ) ) / 2.;

#pragma omp parallel for default(none),shared(dx),reduction(+:sum)
for( int i = 1; i < numSubdivisions - 1; i++ )
{
        double x = A + dx * (float) i;
        double f = Function( x );
        sum += f;
}

sum *= dx;
```

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## Two Reasons Why Reduction is so Much Better in this Case

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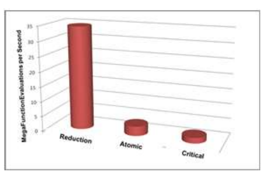
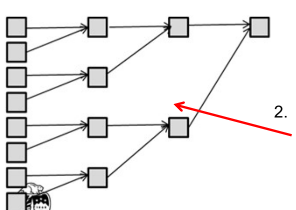


```
#pragma omp parallel for shared(dx),reduction(+:sum)
for( int i = 0; i < numSubdivisions; i++ )
{
        double x = A + dx * (float) i;
        double f = Function( x );
        sum += f;
}
```



1. Reduction secretly creates a temporary private variable for each thread's running ***sum***. Each thread adding into its own running ***sum*** doesn't interfere with any other thread adding into *its* own running ***sum***, and so threads don't need to slow down to get out of the way of each other.
2. Reduction automatically creates a binary tree structure, like this, to add the N running sums in $\log_2 N$ time instead **N** time.

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## O(N) vs. O($\log_2$N)

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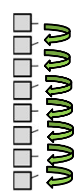
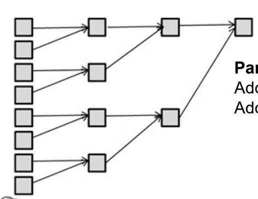
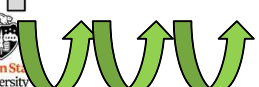
2. Reduction automatically creates a binary tree structure, like this, to add the N running sums in $\log_2 N$ time instead **N** time.



**Serial addition:**
Adding 8 numbers requires 7 steps
Adding 1,048,576 (1M) numbers requires 1,048,575 steps



**Parallel addition:**
Adding 8 numbers requires 3 steps
Adding 1,048,576 (1M) numbers requires 20 steps



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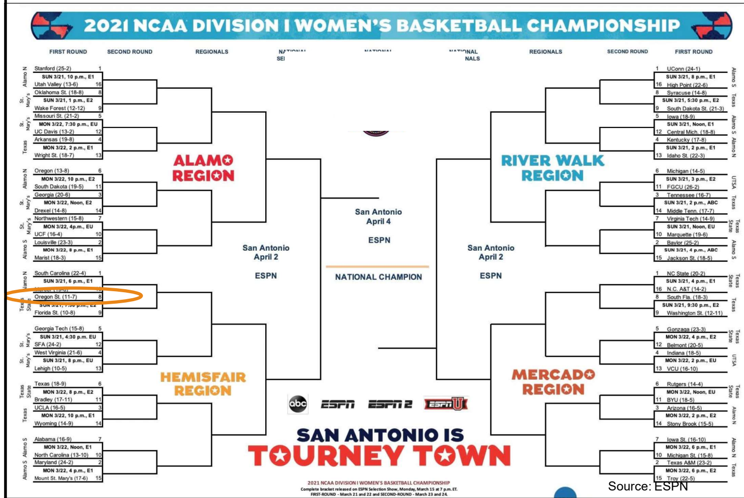
## If You Understand NCAA Basketball Brackets, You Understand Power-of-Two Reduction






Source: ESPN



## Why Not Do Reduction by Creating Your Own *sums* Array, one for each Thread, Like This?



```
float *sums = new float [  omp_get_num_threads( )  ];
for( int i = 0; i < omp_get_num_threads( ); i++ )
            sums[ i ] = 0.;

#pragma omp parallel for private(myPartialSum),shared(sums)
 for( int i = 0; i < N; i++ )
{
      myPartialSum = …

      sums[  omp_get_thread_num( )  ]   +=   myPartialSum;
}

float sum = 0.;
for( int i= 0; i < omp_get_num_threads( ); i++ )
            sum += sums[ i ];

delete [ ] sums;
```

- This seems perfectly reasonable, it works, and it gets rid of the problem of multiple threads trying to write into the same reduction variable.

- The reason we don't do this is that this method provokes a problem called **False Sharing**. We will get to that when we discuss caching.