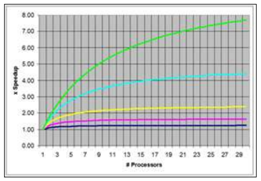
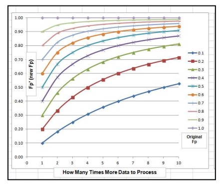
1

Parallel Programming: Speedups and Amdahl's law



Oregon State University
Mike Bailey
mjb@cs.oregonstate.edu



Oregon State University
Computer Graphics

speedups.and.amdahls.law.pptx

mjb – March 21, 2021

1

Definition of Speedup

2

If you are using **n** processors, your ***Speedup$_n$*** is:

$$Speedup_n = \frac{T_1}{T_n}$$

where T_1 is the execution time on **one core** and T_n is the execution time on **n cores**. Note that $Speedup_n$ should be > 1.

And your ***Speedup Efficiency$_n$*** is:

$$Efficiency_n = \frac{Speedup_n}{n}$$

which could be as high as 1., but probably never will be.

Oregon State University
Computer Graphics

mjb – March 21, 2021

2

However, Multicore is not a Free Lunch: Amdahl's Law

3

If you put in *n* processors, you should get *n* times Speedup (and 100% Speedup Efficiency), right? Wrong!

There are always some fraction of the total operation that is inherently sequential and cannot be parallelized no matter what you do. This includes reading data, setting up calculations, control logic, storing results, etc.

If you think of all the operations that a program needs to do as being divided between a fraction that is parallelizable and a fraction that isn't (i.e., is stuck at being sequential), then **Amdahl's Law** says:

$$Speedup_n = \frac{T_1}{T_n} = \frac{1}{\frac{F_{parallel}}{n} + F_{sequential}} = \frac{1}{\frac{F_{parallel}}{n} + (1 - F_{parallel})}$$

This fraction can be reduced by deploying multiple processors.

This fraction can't.

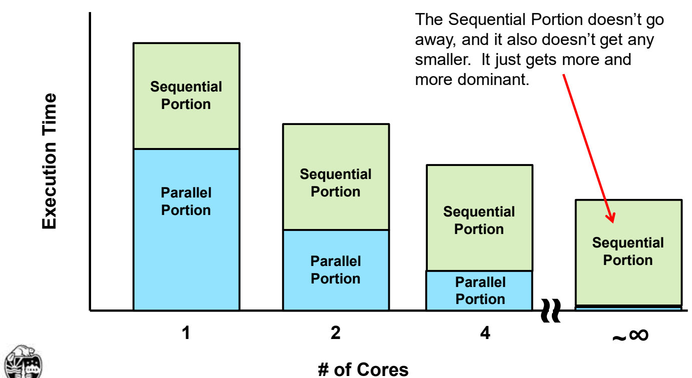
Oregon State University
Computer Graphics

mjb – March 21, 2021

3

A Visual Explanation of Amdahl's Law

4



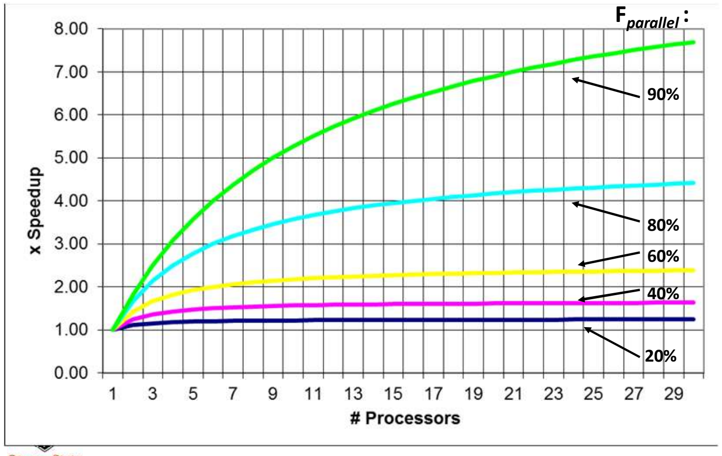
Oregon State University
Computer Graphics

mjb – March 21, 2021

4

SpeedUp as a Function of Number of Processors and $F_{parallel}$

5



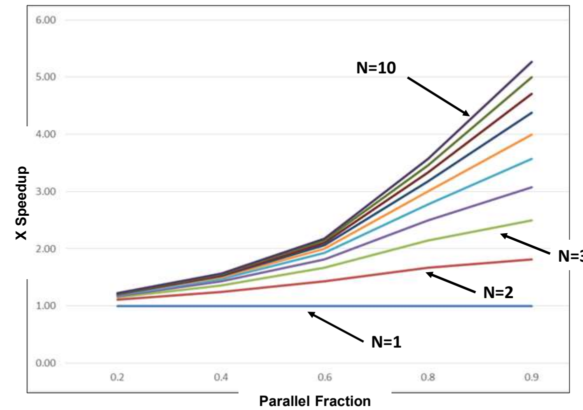
Oregon State University Computer Graphics

mjb – March 21, 2021

5

SpeedUp as a Function of $F_{parallel}$ and Number of Processors

6



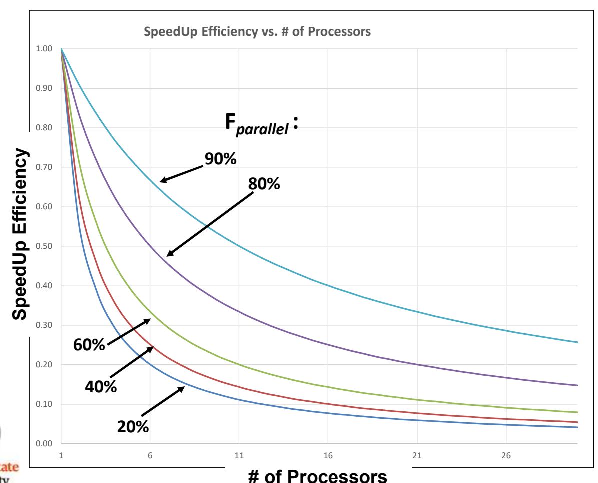
Oregon State University Computer Graphics

mjb – March 21, 2021

6

SpeedUp Efficiency $\left(\frac{S_N}{N}\right)$ as a Function of Number of Processors and $F_{parallel}$

7



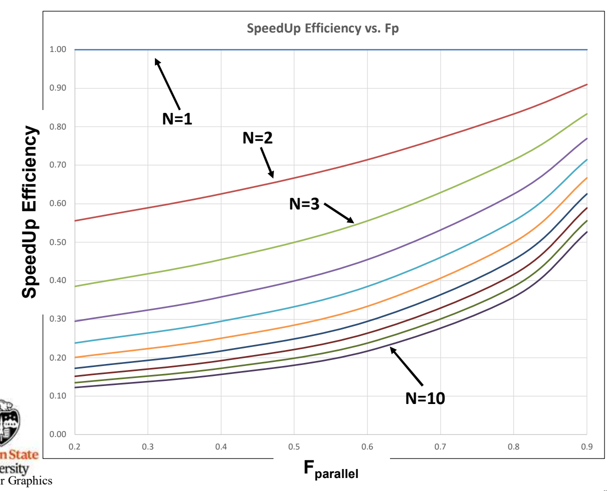
Oregon State University Computer Graphics

mjb – March 21, 2021

7

SpeedUp Efficiency $\left(\frac{S_N}{N}\right)$ as a Function of $F_{parallel}$ and Number of Processors

8



Oregon State University Computer Graphics

mjb – March 21, 2021

8

You can also solve for $F_{parallel}$ using Amdahl's Law if you know your speedup and the number of processors

9

Amdahl's law says:

$$S = \frac{T_1}{T_n} = \frac{1}{\frac{F}{n} + (1-F)} \Rightarrow \frac{1}{S} = \frac{F}{n} + (1-F) = 1 + \frac{F - nF}{n} \Rightarrow \frac{1}{S} - 1 = F\frac{(1-n)}{n}$$

Use this if you know the timing

Use this if you know the speedup

Solving for F:

$$F = \frac{\frac{1}{S} - 1}{\frac{1-n}{n}} = \frac{\frac{T_n}{T_1} - 1}{\frac{1-n}{n}} = \frac{\frac{T_n - T_1}{T_1}}{\frac{1-n}{n}} = \frac{\frac{T_1 - T_n}{T_1}}{\frac{n-1}{n}} = \frac{n(T_1 - T_n)}{T_1(n-1)} = \boxed{\frac{n}{(n-1)}\frac{T_1 - T_n}{T_1}} = \boxed{\frac{n}{(n-1)}\left(1 - \frac{1}{Speedup}\right)}$$

If you've got several (n,S) values, you can take the average (which is actually a least squares fit):

$$F_i = \frac{n_i}{(n_i - 1)}\frac{T_1 - T_{n_i}}{T_1}, i = 2..N$$

$$\overline{F} = \frac{\sum_{i=2}^{N} F_i}{N-1}$$

note that when i=1, $T_{ni} = T_1$

Oregon State University Computer Graphics

mjb – March 21, 2021

9

Amdahl's Law can also give us the Maximum Possible SpeedUp

10

Note that these fractions put an upper bound on how much benefit you will get from adding more processors:

$$\max Speedup = \lim_{n \to \infty} Speedup = \frac{1}{F_{sequential}} = \frac{1}{1 - F_{parallel}}$$

Fparallel	maxSpeedup
0.00	1.00
0.10	1.11
0.20	1.25
0.30	1.43
0.40	1.67
0.50	2.00
0.60	2.50
0.70	3.33
0.80	5.00
0.90	10.00
0.95	20.00
0.99	100.00

Oregon State University Computer Graphics

mjb – March 21, 2021

10

A More Optimistic Take on Amdahl's Law: The Gustafson-Baris Observation

11

Gustafson observed that as you increase the number of processors, you have a tendency to attack larger and larger versions of the problem. He also observed that when you use the same parallel program on larger datasets, the parallel fraction, F_p, increases.

Let P be the amount of time spent on the parallel portion of an original task and S spent on the serial portion. Then

$$F_p = \frac{P}{P+S}$$

Parallel Time

Serial Time

or

$$S = \frac{P - PF_p}{F_p}$$

Without loss of generality, we can set P=1 so that, really, S is now a fraction of P. We now have:

$$S = \frac{1 - F_p}{F_p}$$

Oregon State University Computer Graphics

mjb – March 21, 2021

11

A More Optimistic Take on Amdahl's Law: The Gustafson-Baris Observation

12

We know that if we multiply the amount of data to process by N, then the amount of parallel work becomes NP. Surely the serial work must increase too, but we don't know how much. Let's say it doesn't increase at all, so that we know we are getting an upper bound answer.

In that case, the new parallel fraction is: $F_p' = \frac{P'}{P' + S} = \frac{NP}{NP + S}$

And substituting for P (=1) and for S, we have:

$$F_p' = \frac{N}{N+S} = \frac{N}{N + \frac{1 - F_p}{F_p}}$$

Oregon State University Computer Graphics

mjb – March 21, 2021

12

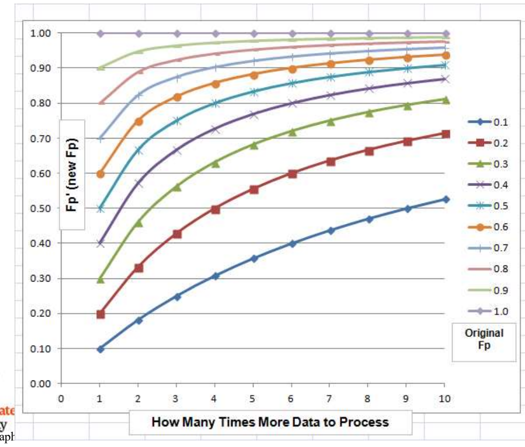
A More Optimistic Take on Amdahl's Law: The Gustafson-Baris Observation

If we tabulate this, we get a table of F_p' values:

Original Fp	How Many Times More Data to Process → 1	2	3	4	5	6	7	8	9	10
0.1	0.10	0.18	0.25	0.31	0.36	0.40	0.44	0.47	0.50	0.53
0.2	0.20	0.33	0.43	0.50	0.56	0.60	0.64	0.67	0.69	0.71
0.3	0.30	0.46	0.56	0.63	0.68	0.72	0.75	0.77	0.79	0.81
0.4	0.40	0.57	0.67	0.73	0.77	0.80	0.82	0.84	0.86	0.87
0.5	0.50	0.67	0.75	0.80	0.83	0.86	0.88	0.89	0.90	0.91
0.6	0.60	0.75	0.82	0.86	0.88	0.90	0.91	0.92	0.93	0.94
0.7	0.70	0.82	0.88	0.90	0.92	0.93	0.94	0.95	0.95	0.96
0.8	0.80	0.89	0.92	0.94	0.95	0.96	0.97	0.97	0.97	0.98
0.9	0.90	0.95	0.96	0.97	0.98	0.98	0.98	0.99	0.99	0.99
1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

A More Optimistic Take on Amdahl's Law: The Gustafson-Baris Observation

Or, graphing it:



A More Optimistic Take on Amdahl's Law: The Gustafson-Baris Observation

We can also turn F_p' into a Maximum Speedup: